647510 **BSc3MatC301x** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov - 2023 [NEW COURSE]
Mathematics-M301(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q-1 **A) ATTEMPT ANY TWO:** **(10)**

1) Define: Vector Space.
Prove that in a vector space V, $(-v)$ is unique, $\forall v \in V$.

2) Define: Subspace of a vector space.
Prove that a non-empty subset W of a vector space V is a subspace of V *iff* $\alpha w_1 + \beta w_2 \in W, \forall\, w_1, w_2 \in W, \forall \alpha, \beta \in F = \mathbb{R}$.

3) Define: Linearly Independent Set & Linearly Dependent Set
Explain: A non-empty subset of a vector space containing two or more vectors in which one vector is the zero vector θ, is always linearly dependent.

B) ATTEMPT ANY ONE : **(04)**

1) Show that the set $V = \{(x, y) \in \mathbb{R}^2 \ / \ y > 0\}$ is a vector space over $F = \mathbb{R}$ under the vector addition and scalar multiplication defined as follows:

$$(a, b) + (c, d) = (a + c, bd), \forall (a, b), (c, d) \in V$$

and

$$\alpha(a, b) = (\alpha a, b^\alpha), \forall (a, b) \in V, \forall \alpha \in F = \mathbb{R}.$$

2) Check linear dependence of the following sets:
(i) $\{e^{2x}, e^{4x}, e^{6x}\}$
(ii) $\{\sin(-x), \cos(-x), \sin\left(\frac{\pi}{6} - x\right)\}$

Q-2 **ATTEMPT ANY TWO:** **(10)**

1) State and prove the existence theorem for a basis of a finite dimensional vector space.

2) Let V be a finite dimensional vector space.
Then, prove that for every subspace W_1 of V, there exists another subspace W_2 of V such that $V = W_1 \oplus W_2$.

3) If W_1 & W_2 be any two subspaces of a finite dimensional vector space V, then prove that

$$Dim(W_1 + W_2) = Dim(W_1) + Dim(W_2) - Dim(W_1 \cap W_2)$$

B) ATTEMPT ANY ONE: **(04)**

1) Show that the set $\{1 - x, 1 + x, 1 - x^2\}$ is a basis of $P_2(\mathbb{R})$.
Also find the coordinates of the vector $1 - 2x + 7x^2$ w.r.t. this basis.

2) Let $W_1 = \{\, (a, b, c, d) \ / \ a + c - d = 0\}$ & $W_2 = \{\, (a, b, c, d) \,/\, a + 2b = 0\}$ are subspaces of the vector space $\mathbb{R}^4$.
Find $Dim(W_1), Dim(W_2)$ & $Dim(W_1 \cap W_2)$. Also, deduce that $W_1 + W_2 = \mathbb{R}^4$.

Q-3 A) ATTEMPT ANY TWO: **(10)**

1) Define: Gradient

If ϕ and ψ are differentiable scalar functions, then prove the followings:

(i)$\nabla(\phi\psi) = \phi\nabla(\psi) + \psi\nabla(\phi)$ (ii) $\nabla\left(\frac{\phi}{\psi}\right) = \frac{\psi\nabla(\phi)-\phi\nabla(\psi)}{\psi^2}$

2) Define: (i) Divergence (ii) Curl

If $\bar{f}$ be a function whose second order derivative exists, then show that

$div\left(Curl(\bar{f})\right) = 0.$

3) Define: Laplace Operator

Prove that, $\nabla^2 f(r) = f''(r) + \frac{2}{r}f'(r)$ Where $r = |\bar{r}|, \bar{r} = (x, y, z).$

B) ATTEMPT ANY ONE: **(04)**

1) If $(x, y, z) = x^2y + xy^2 + z^2$, then find $\nabla\phi$ and find unit normal at $(1,2,3)$.

2) Prove that $\frac{1}{r}$ satisfies the Laplace Equation and also prove that

$curl\left(grad(r^n)\right) = \theta.$

Q-4 A) ATTEMPT ANY TWO: **(10)**

1) Define: Double Integration

Explain evaluation of double integrals and also explain change of order of integration.

2) Define: Triple Integration

Explain change of variables in cylindrical variables and spherical variables in triple integration.

3) Evaluate: $\iint_R \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\,dxdy$; where R is the unit circle.

B) ATTEMPT ANY ONE: **(04)**

1) Evaluate: $\iint (x+y)^2 dxdy$ over R ,

whrere R is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

2) Evaluate: $\int_0^a \int_0^b \int_0^c (x+y+z)dxdydz.$

Q-5 A) ATTEMPT ANY TWO: **(10)**

1) State and prove the Green's theorem.

2) State Gauss divergence theorem and verify the theorem for $F = (4x, -2y^2, z^2)$ where R is $x^2 + y^2 = 4, z = 0$ & $z = 3$.

3) Prove that $\beta(p, q) = \frac{\lceil p \lceil q}{\lceil(p+q)}$, $where\ p > 0$ & $q > 0$.

B) ATTEMPT ANY ONE: **(04)**

1) Verify the Stoke's theorem for $F = (2x - y, -yz^2, -y^2z)$ & the surface of integration S is the upper half of the unit sphere.

2) Find the values of $\left\lceil\frac{1}{4}\right.\left\lceil\frac{3}{4}\right.$

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